## Password security

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## Password: Kinds of passwords

- Password
- A string of characters: A,B,C,...d,e,f,...1,2,3...!,",@,...
- PIN-code
- A string of numbers
- Pass phrase
- A sentence
- Associative and cognitive passwords
- Answers to the questions
- Associative, cue words
- Black: white, strawberry: blueberry, dad: mum, day: night etc.
- Cognitive
- What is your second name? How many cats do you have? Which chocolate you like best?
- Pass face, pass image


## Password: Password space - S

S is the total set of all passwords

- Size of $S$ is denoted by s
- 4-digit PIN codes: $s=|S|=10^{4}$
- 6 character passwords:
- $\mathrm{s}=26^{6}$
- $S=52^{6}$
- $S=62^{6}$
- $S=94^{6}$


## Password: <br> The art of counting

- Number of possibilities with one dice: 6
- Number of possibilities with two dices:
- Unordered: 21
- Ordered: 36
- Number of 5 letter combinations: $26^{5}$
. Including capitals: $52^{5}$
- Including numbers: $62^{5}$
- All keyboard symbols: $94^{5}$


## Password: Combinatorics - 1

- We will count the number of 6 character passwords
- All is possible: letters, capitals, numbers and special characters
- If no restriction, then we have $94^{6}$ possible passwords
- On the next slides we will introduce specific restrictions


## Password: Combinatorics - 2

- At least 1 number?
- Total number of 6 character passwords: $94^{6}$
- Number of 6 character passwords without numbers: $84^{6}$
- Answer: $94^{6}-84^{6}=338.571 .749 .440$
- Trick: All - those that are wrong


## Password: Combinatorics - 3

- Have 6 different characters?
- First character: 94 possibilities
- Second character: (94-1) possibilities
- Third character: (94-2) possibilities
- Answer: 94*93...*89 = 586.236.072.240 =
- Trick: Count every time what is still possible


## Password: Combinatorics - 4

- At least 1 capital and 1 number?
- No restrictions: $94^{6}$
- No capitals: 686
- No numbers: $84^{6}$
- No capitals and no numbers: $58^{6}$
- Answer: $94^{6}-68^{6}-84^{6}+58^{6}=277.772 .959 .360=$ 238,02
- Trick: All - wrong ones + those subtracted twice!


## Password: Combinatorics - 5

- Exactly 1 number?
- Choose position where the number will be: 6 possibilities
- Number on that position: 10 possibilities
- All other 5 positions: (94-10) possibilities
- Answer: $(6 * 10)$ * $845=250.927 .165 .440$ Trick: Place number first.


## Password: <br> Combinatorics - 6

- Exactly 1 number and exactly 1 capital?
- Choose position for the number: 6 possibilities
- Number on that position: 10 possibilities
- Choose position for the capital: (6-1) possibilities
- Capital on that position: 26 possibilities
- All other 4 positions: (94-10-26) possibilities
- Answer: $\left(6^{*} 10\right)$ * $\left(5^{*} 26\right) * 58^{4}=88.268 .668 .800$
- Trick: Place number and capital first


## Password: Combinatorics - 7

- Exactly 2 numbers?
- Choose 2 positions for the numbers:
$6 * 5 / 2=15$ possibilities
- Numbers on those position: 10 possibilities
- All other 4 positions: (94-10) possibilities
- Answer: $15 * 10^{2}$ * $84^{4}=74.680 .704 .000=$


## Password: Combinatorics - 8

- Choose 2 positions for the numbers gives 15 possibilities. Why?
- "Choose m out of $n$ ":

$$
\begin{gathered}
n!/(m!*(n-m)!) \\
=k!=1^{*} 2^{*} \ldots *(k-1)^{*} k
\end{gathered}
$$

- "Choose 2 out of 6": 6!/(2!*4!) = 15


## Password: Probabilities

- What is the probability that a random password of 6 characters has no number in it?
- Answer: $84^{6} / 94^{6}=(84 / 94)^{6}=0,509$
- So approximately have of the 6 character passwords does not have a number in it!
- In general is the probability equal to the size of set of correct answers divided by the total number of answers.


## Password: <br> Statistics - Introduction

- Let $\mathbf{x}=\left(\mathrm{x}_{1}, \mathrm{x}_{2}, \ldots, \mathrm{x}_{\mathrm{n}}\right)$ and $\mathbf{y}=\left(\mathrm{y}_{1}, \mathrm{y}_{2}, \ldots, \mathrm{y}_{\mathrm{n}}\right)$ be two equally long sequence of numbers.
- Let $p_{i}$ be the probability that occasion $x_{i}$ occurs.
- $p_{1}+p_{2}+\ldots+p_{n}=1$


## Password: <br> Statistics - Mean $\mu$

The mean of $\mathbf{x}$ is the weighted average of the values of $\mathbf{x}$. The weights are the probabilities.

- Also called "Expected value"
- $E(\mathbf{x})=\mu_{\mathbf{x}}$
- The mean $\mu_{\mathbf{x}}$ of $\mathbf{x}$ is defined as:

$$
\mu_{\mathrm{x}}=\mathrm{p}_{1} \mathrm{x}_{1}+\mathrm{p}_{2} \mathrm{x}_{2}+\ldots+\mathrm{p}_{\mathrm{n}} \mathrm{x}_{\mathrm{n}}
$$

## Password: <br> Statistics - Mean $\mu$-example

- Values of a dice: $\mathbf{x}=(1,2,3,4,5,6)$
- True dice: $\mathbf{p}=(1 / 6,1 / 6,1 / 6,1 / 6,1 / 6,1 / 6)$

$$
\mu_{\mathbf{x}}=(1+2+3+4+5+6) / 6=3.5
$$

## Password: <br> Statistics - Variance $\sigma^{2}$

The variance is a measure of how much the members of $\mathbf{x}$ are scattered around their mean.

The variance $\sigma_{\mathbf{x}}{ }^{2}$ of $\mathbf{x}$ is defined as:

$$
\begin{aligned}
\sigma_{\mathbf{x}}^{2}= & V(\mathbf{x})=E\left(\mathbf{x}-\mu_{\mathbf{x}}\right)^{2}= \\
& =E\left(\mathbf{x}^{2}\right)-2 \mu_{\mathbf{x}} \mathrm{E}(\mathbf{x})+\left(\mu_{\mathbf{x}}\right)^{2}= \\
& =E\left(\mathbf{x}^{2}\right)-\left(\mu_{\mathbf{x}}\right)^{2}
\end{aligned}
$$

## Password: Statistics - Covariance $\sigma_{\mathbf{x y}}$

- We use covariance to measure similarity between $\mathbf{x}$ and $\mathbf{y}$.
- $\sigma_{\mathbf{x y}}=\mathrm{E}\left(\left(\mathbf{x}-\mu_{\mathbf{x}}\right) *\left(\mathbf{y}-\mu_{\mathbf{y}}\right)\right)$


## Password:

## Statistics - Correlation $\rho_{\mathbf{x y}}$

- $\rho_{\mathbf{x y}}=\sigma_{\mathbf{x y}} /\left(\sigma_{\mathbf{x}}^{*} \sigma_{\mathbf{y}}\right)$
- If $\rho_{\mathbf{x y}}=0$ then $\mathbf{x}$ and $\mathbf{y}$ are uncorrelated.
- The larger $\left|\rho_{\mathbf{x y}}\right|$ is, the more $\mathbf{x}$ and $\mathbf{y}$ are correlated.
- Sign of $\rho_{\mathbf{x y}}$ tells something about direction of correlation


## Password: Entropy - h

Entropy h is a measure of the randomness

- Entropy $h$ is the number of bits needed to describe the members of $S$
- In formula:
- $\mathrm{h}=\log _{2}(\mathrm{~s})$
- Assumption: all passwords are equally likely


## Password: Examples of entropy

- 4-digit PIN code:
- $\mathrm{s}=10^{4}$
- $\mathrm{h}=\log _{2}\left(10^{4}\right)=13,3$
- 6 character password
- $\mathrm{S}=94^{6}$
- $h=\log _{2}\left(94^{6}\right)=39,3$


## Password: Entropy - more complicated

- Let $S=\left\{s_{1}, s_{2}, \ldots, s_{s}\right\}$
- Let $P=\left\{p_{1}, p_{2}, \ldots, p_{s}\right\}$, where $p_{i}$ is the probability someone uses password $\mathrm{s}_{\mathrm{i}}$
- Entropy is now defined as:
- $h=-p_{1} \log \left(p_{1}\right)-p_{2} \log \left(p_{2}\right)-\ldots-p_{s} \log \left(p_{s}\right)$


## Password: Entropy - more complicated

- If $p_{i}=1 / s$ for all $i$ then:

$$
\begin{aligned}
\mathrm{h} & =-1 / \mathrm{s} \log (1 / \mathrm{s})-\ldots-1 / \mathrm{s} \log (1 / \mathrm{s})= \\
& =-\mathrm{s}^{*} 1 / \mathrm{s} * \log (1 / \mathrm{s}) \\
& =-\log (1 / \mathrm{s})=\log (\mathrm{s})
\end{aligned}
$$

- So definitions are consistent


## Password: <br> Good Properties

. Hard to guess: do not use names, dates, telephone numbers, etc.

- Easy to remember: no need to write it down or share with other persons
- Private: otherwise no authentication possible
- Secret: owner is the only one who knows it


## Password: Attacks

- Dictionary attack
- Not fooled by
- Capitals
- Change of letters into numbers
- Permutations
- What can we do?

Password:

## To not do list - 1

- PW based on user's account name
- PW which match a word (or reversed word) in a dictionary, regardless if some or all of the letters are capitalized
- PW which match a word in a dictionary with an arbitrary letter turned into a control character

Password:

## To not do list - 2

- PW which are simple conjugations of a dictionary word (i.e. plurals, adding "ing" or "ed" to end of word, etc.)
- PW which do not use mixed upper and lower case, or mixed letters and numbers, or mixed letters and punctuation


## Password:

## To not do list - 3

. PW base on user's initials or given name

- PW which match a dictionary word with letters replaced by numbers (eg '3' for 'e')
- PW which are patterns from the keyboard (eg. "aaaaa" or "qwerty")
- PW which only consist of numbers


## Password: The PROBLEM!

- We have limited memory
- Can only remember $7 \pm 2$ totally random symbols
- Even more problems when
- We have multiple passwords
- We need to change passwords regularly


## Password: What can we do - part 1 ?

- Pass phrase
- Yesterday I watched a nice program on television.
- YIwanpot or Y1wanp0t
- Use events on news or personal events when forced to change regularly


## Password:

 What can we do - part 2?- Encryption
- Shift every character fixed number of positions
- Shift every character by increasing number of positions
- http://geodsoft.com/cgi-bin/pwcheck.pl


## Password: Pass faces and images

- It is easier to recognize then to remember.
- Setup:
- Memorize a set of selected or given pictures
- Authentication:
- Recognize memorized pictures


## Password: Pass faces

- Five faces are presented and need to be memorized
- Five $4 x 4$ grids are presented each containing 1 memorized image


## Password: Pass images

- p (random) images selected and remembered
- n images presented containing m selected images
- Vary value of m during authentication
- Present more challenges


## Password: References

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